FLOW OF AN ANOMALOUSLY VISCOUS FLUID FILM IN A CENTRIFUGAL FORCE FIELD

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We consider the flow of an anomalously viscous film along the surface of a curvilinear attachment. The deformation behavior of the liquid is described by an exponential relation.

Centrifugal apparatuses that employ the film flow of liquids are widely used for carrying out hydrodynamic and heat and mass transfer processes: dispersion for depositing protective coatings, drying and cooling by spraying, granulation, molecular distillation, mixing in a liquid medium, rectification, absorption, heating, vaporization, etc., as well as for conducting chemical reactions [1-5]. Basic to these apparatuses is a rotor consisting of a shaft with attachments in the form of a flat disk, cone, sphere or other curvilinear body, cylindrical tube, or their combinations. A comprehensive functional analysis of rotor designs for centrifugal apparatuses and attachments is made in [6].

Treated in centrifugal apparatuses are materials characterized by various rheological properties. The hydrodynamics of film flow in a centrifugal field has been studied predominantly for viscous fluids. The flow of an anomalously viscous fluid, just as of a viscous fluid, has been investigated for a flat disk and a cone [6-8]. Different rheological models have been used, namely, power law [7], Williamson's model [8], and ordinary and modified Cross's models [6].

Anomalously viscous fluids treated in centrifugal apparatuses are usually characterized by high values of viscosity; therefore, no wave formation has been discovered in them. Possible ruptures of the fluid film and conditions for attaining the minimum wetting density were considered in [9].

We examine a steady-state isothermal anomalously viscous fluid flowing in the form of a thin continuous laminar film (without wave formation) along a rotating curvilinear attachment.

The following assumptions are adopted: inertia forces are small; velocity components relate as $v_1 \sim v_2 \gg v_3$; the coordinate system q_1, q_2, q_3 is orthogonal curvilinear and rotates together with the surface; the flow geometry is such that q_1, $q_2 \gg q_3$; the flow is axisymmetric, i.e., $\partial/\partial q_2 = 0$; the physical parameters change along q_3 much more rapidly than along q_1 and q_2; the attachment has the form of a surface of second-order of rotation; the rheological equation of state can be represented in the form of an exponential relation ("power law"). The deviator part of the stress tensor is of the form

$$D_p = 2\mu_{ef}^* D_\varepsilon ,$$

while the magnitude of the generalized effective viscosity is

$$\mu_{ef}^* = KE^{n-1} .$$

Then, from the general differential equations of motion in an orthogonal curvilinear coordinate system [10] we obtain

$$F_1 - \frac{1}{\rho H_1} \frac{\partial p}{\partial q_1} + \frac{K}{\rho} E^{n-1} M_1 + \frac{K}{\rho H_3} \left[\frac{H_1}{H_3} \frac{\partial}{\partial q_3} \left(\frac{v_1}{H_1} \right) \right] \frac{\partial}{\partial q_3} (E^{n-1}) = 0 , \qquad (1)$$
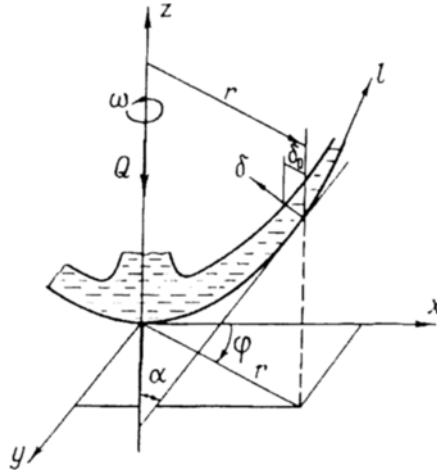
Fig. 1. Diagram of fluid flow.

$$F_2 + \frac{K}{\rho} E^{n-1} M_2 + \frac{K}{\rho H_3} \left[\frac{H_2}{H_3} \frac{\partial}{\partial q_3} \left(\frac{v_2}{H_2} \right) \right] \frac{\partial}{\partial q_3} (E^{n-1}) = 0 , \tag{2}$$

$$F_3 - \frac{1}{\rho H_3} \frac{\partial p}{\partial q_3} + \frac{K}{\rho} E^{n-1} M_3 + \frac{2K}{\rho H_3^2} \frac{v_1}{H_1} \frac{\partial H_3}{\partial q_3} \frac{\partial}{\partial q_3} (E^{n-1}) = 0 . \tag{3}$$

Here

$$M_i = f \left(H , \frac{\partial v}{\partial q} , \frac{\partial H}{\partial q} , \frac{\partial^2 v}{\partial q^2} , v \right) .$$

Suppose the curvilinear attachment has the form shown in Fig. 1. We select a coordinate system such that the boundaries of the attachment are among the coordinate surfaces. Thus, we adopt the following system of coordinates: $q_1 = l$; $q_2 = \varphi$; $q_3 = \delta$. Lame's coefficients are defined as $H_l = 1$; $H_\varphi = r$; $H_\delta = 1$. The shape of the attachment is characterized by the quantities $\sin \alpha$ and $\cos \alpha$, which are equal to:

$$\sin \alpha = \frac{dr}{dl} = 1 \Big/ \left[1 + \left(\frac{\partial z}{\partial r} \right)^2 \right]^{1/2} , \quad \cos \alpha = \frac{dz}{dl} = \frac{dz}{dr} \Big/ \left[1 + \left(\frac{\partial z}{\partial r} \right)^2 \right]^{1/2} . \tag{4}$$

The initial system of equations has the form

$$F_l - \frac{1}{\rho} \frac{\partial p}{\partial l} + \frac{K}{\rho} \frac{\partial}{\partial \delta} \left[E^{n-1} \frac{\partial v_l}{\partial \delta} \right] = 0 , \tag{5}$$

$$F_\varphi + \frac{K}{\rho} \frac{\partial}{\partial \delta} \left[E^{n-1} \frac{\partial v_\varphi}{\partial \delta} \right] = 0 , \tag{6}$$

$$F_\delta - \frac{1}{\rho} \frac{\partial p}{\partial \delta} = 0 . \tag{7}$$

Here

$$E = + \left[\left(\frac{\partial v_l}{\partial \delta} \right)^2 + \left(\frac{\partial v_\varphi}{\partial \delta} \right)^2 \right]^{1/2} ,$$

$$F_l = (\omega^2 r - 2\omega v_\varphi) \sin \alpha \,, \quad F_\varphi = 2\omega v_l \sin \alpha \,, \quad F_\delta = -(\omega^2 r - 2\omega v_\varphi) \cos \alpha \,. \tag{8}$$

The boundary conditions are

$$\delta = \delta_0 : \; p = p_0 \,, \quad \frac{\partial v_l}{\partial \delta} = 0 \,, \quad \frac{\partial v_\varphi}{\partial \delta} = 0 \,; \quad \delta = 0 : \; v_l = 0 \,, \quad v_\varphi = 0 \,. \tag{9}$$

Solution of Eq. (7) under boundary conditions (9) permits us to determine the order of $\partial p / \partial l$. When $\alpha \le 75 - 80^0$, the ratio of $(1/\rho)(\partial p / \partial l)$ to the other terms has the form δ_0 / l, therefore, it can be neglected.

Direct integration of (5) and (6) involves mathematical difficulties. Because of this, we apply approximate methods of solution.

Method of Averaging of Body Forces. This method is based on averaging of the body forces over the thickness of the film layer:

$$F_{ic} = \frac{1}{\delta_0} \int_0^{\delta_0} F_i d\delta \,,$$

Integration of Eqs. (5) and (6) under boundary conditions (9) with allowance for E yields:

$$\left(\frac{K}{\rho}\right)^{1/n} (F_{l\,m}^2 + F_{\varphi\,m}^2)^{\frac{n-1}{2n}} \frac{\partial v_l}{\partial \delta} = F_{l\,m} (\delta_0 - \delta)^{1/n} \,, \tag{10}$$

$$\left(\frac{K}{\rho}\right)^{1/n} (F_{l\,m}^2 + F_{\varphi\,m}^2)^{\frac{n-1}{2n}} \frac{\partial v_\varphi}{\partial \delta} = F_{\varphi\,m} (\delta_0 - \delta)^{1/n} \,. \tag{11}$$

Solution of Eqs. (10) and (11) permits us to find dependences for the velocity profile:

$$\frac{v_l}{v_{l_0}} = \frac{v_\varphi}{v_{\varphi 0}} = \left[1 - \left(1 - \frac{\delta}{\delta_0}\right)^{\frac{1+n}{n}}\right] \,, \tag{12}$$

where

$$v_{i_0} = F_{i\,m} A_1 \,; \quad A_0 = \left(\frac{\rho}{K}\right)^{1/n} \left(\frac{n}{n+1}\right) \delta_0^{\frac{1+n}{n}} \Big/ (F_{l\,m}^2 + F_{\varphi\,m}^2)^{\frac{n-1}{2n}} \,.$$

Now, we average V_l and V_φ over the layer thickness and obtain

$$v_{l\,m} = \left(\frac{n+1}{2n+1}\right) v_{l_0} \,, \quad v_{\varphi\,m} = \left(\frac{n+1}{2n+1}\right) v_{\varphi 0} \,.$$

Using the flow measurement equation

$$Q = 2\pi r \delta_0 v_{l\,m} \tag{13}$$

and introducing dimensionless velocities in the form

$$\bar{v}_{i\,m} = 4 v_{i\,m} / \omega r \,,$$

we obtain from Eq. (12):

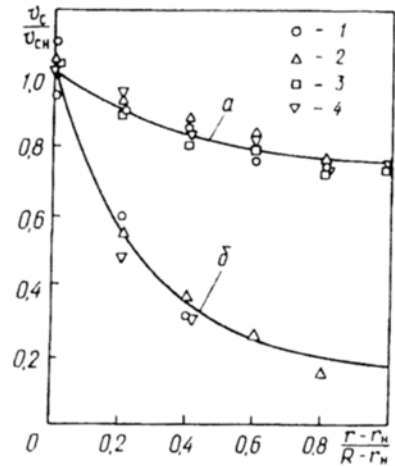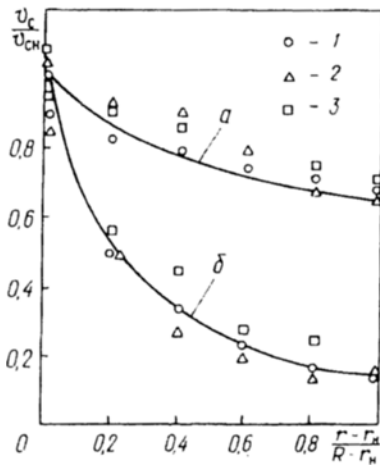$$\bar{v}_{l\,m} = A_1 \left[1 - (1 - \bar{v}_{l\,m}^2)^{1/2}\right]^{\frac{n+1}{4n+2}} \tag{14}$$

Fig. 2. Function $v_m/v_{m\ in} = f((r - r_{in})/(R - r_{in}))$ for a 1.0% aqueous KMTs-300 solution ($K = 0.06$ Pa·sec^n, $n = 0.80$): a) $v_{l\ m}/v_{l\ m\ in}$; b) $v_{\varphi\ m}/v_{\varphi\ m\ in}$; 1, $Q = 2 \cdot 10^{-5}$ [m^3/sec], $\omega = 130$ [sec^{-1}], $\alpha = 90°$; 2, $Q = 3 \cdot 10^{-5}$ [m^3/sec], $\omega = 260$ [sec^{-1}], $\alpha = 90°$; 3, $Q = 2.5 \cdot 10^{-5}$ [m^3/sec], $\omega = 240$ [sec^{-1}], $\alpha = 60°$.

Fig. 3. Function $v_m/v_{m\ in} = f((r - r_{in})/(R - r_{in}))$ for a 2.0% aqueous PAA solution ($K = 4.2$ Pa·sec^n, $n = 0.52$): a) $v_{l\ m}/v_{l\ m\ in}$; b) $v_{\varphi\ m}/v_{\varphi\ m\ in}$; 1, $Q = 2 \cdot 10^{-5}$ [m^3/sec], $\omega = 420$ [sec^{-1}], $\alpha = 90°$; 2, $Q = 4.8 \cdot 10^{-5}$ [m^3/sec], $\omega = 315$ [sec^{-1}], $\alpha = 90°$; 3, $Q = 8 \cdot 10^{-6}$ [m^3/sec], $\omega = 131$ [sec^{-1}], $\alpha = 60°$; 4, $Q = 1.5 \cdot 10^{-5}$ [m^3/sec], $\omega = 210$ [sec^{-1}], $\alpha = 60°$.

$$\bar{v}_{\varphi\ m} = \left[1 - (1 - \bar{v}_{l\ m}^2)^{1/2} \right], \tag{15}$$

where

$$A_1 = \left[\frac{\rho}{K} \left(\frac{n}{2n + 1} \right) \left(\frac{Q}{\pi} \right)^{n+1} \frac{2^{\frac{5n+1}{2}} \sin \alpha}{\omega^{2n-1} r^{3n+1}} \right]^{\frac{1}{2n+1}}.$$

Dependences (14), (15), and (13) allow determination of the basic hydrodynamic parameters of the film of the fluid flowing along the surface of the rotating attachment (rotor).

Method of Integral Relations. As is known, the method is based on the replacement of equations by corresponding integral relations.

Let the profile of v_l and v_φ have the form described by Eq. (12). The same profile was obtained in [10] for v_l for a one-dimensional fluid flow.

For the Karman case (the general case belongs to Golubev) the following equations were obtained:

$$\omega^2 r \sin \alpha \int_0^{\delta_0} d\delta - 2\omega \sin \alpha \int_0^{\delta_0} v_m d\delta + \frac{K}{\rho} \int_0^{\delta_0} \frac{\partial}{\partial \delta} \left[E^{n-1} \frac{\partial v_l}{\partial \delta} \right] d\delta = 0, \tag{16}$$

$$2\omega \sin \alpha \int_0^{\delta_0} v_l d\delta + \frac{K}{\rho} \int_0^{\delta_0} \frac{\partial}{\partial \delta} \left[E^{n-1} \frac{\partial v_\varphi}{\partial \delta} \right] d\delta = 0. \tag{17}$$

Equations (16) and (17) with allowance for Eq. (13), after the introduction of dimensionless relations, yield

$$\overline{v}_{l\,\mathrm{m}}^{\,n}\left[1 - (1 - \overline{v}_{l\,\mathrm{m}}^{\,2})^{1/2}\right]^{\frac{n+1}{2}} = A_2,\tag{18}$$

$$\overline{v}_{\varphi\,\mathrm{m}}\left[1 - (1 - \overline{v}_{l\,\mathrm{m}}^{\,2})^{1/2}\right].\tag{19}$$

Here

$$A_2 = \left(\frac{\rho}{K}\right)\left(\frac{n}{2n+1}\right)^n\left(\frac{Q}{\pi}\right)^{n+1}\frac{2^{\frac{5n+1}{2}}\sin\alpha}{\omega^{2n-1}r^{3n+1}} = \left(\frac{n}{2n+1}\right)^{n-1}A_1^{2n+1}.$$

The basic parameters of the process are determined from relations (18), (19), and (13).

We easily recognize that Eqs. (14) and (18) are identical, and that Eqs. (15) and (19) coincide. When $0 < \overline{v}_{l\,\mathrm{m}} < 0.2$, the lag can be ignored. Thus, the approximate solutions based on averaging of the body forces over the layer thickness and on the application of integral relations (the Karman case) give identical results.

To check the dependences obtained, we carried out experiments. The test setup and the experimental procedure are described in [11].

Figures 2 and 3 present some characteristic experimental data on velocities. The theoretical curves are plotted with allowance for the lag velocity. As we can see from the figures, the magnitudes of both the meridional and tangential velocities decrease with an increase in the radius of the attachment (the tangential velocity decreases more intensity, especially at small radii).

The results obtained show the sufficient accuracy of the methods proposed for determining the basic flow parameters.

NOTATION

D_p, D_ε, deviator of the stress tensor and strain rate; E, intensity of shear rate; K and n, rheological constants of fluid; F_i ($i = 1, 2, 3$) projections of unit body forces onto corresponding axes; p and p_0, current and surface (atmospheric) pressure; r, distance from surface to rotation axis; δ_0, film thickness; ρ, density; ω, angular velocity of attachment.

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